

**Class: XII      MODEL EXAM-1-Nov-19-Mathematics      Marks: 80**  
**Date: 18.11.19      Time: 3 hrs**

**General Instructions:**

- (i) All the questions are compulsory.
- (ii) The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- (iii) Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- (iv) There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted.

**SECTION-A**

**Choose the most suitable answer:**

1. The value of  $\int_{-2}^2 (x^3 + x) dx$   
(a) 1/2      (b) 12      (c) 0      (d) 6
2.  $\int_1^e \log x dx =$   
(a) 1      (b)  $e - 1$       (c)  $e + 1$       (d) 0
3.  $\int_0^{2a} f(x) dx =$   
(a)  $\int_0^a f(x) dx + \int_0^a f(a-x) dx$       (b)  $\int_0^a f(x) dx + \int_0^a f(2a-x) dx$       (c)  $\int_0^a f(x) dx + \int_0^{2a} f(a-x) dx$   
(d)  $\int_0^a f(x) dx - \int_0^a f(a-x) dx$

4. The area bounded by the ellipse  $\frac{x^2}{3^2} + \frac{y^2}{16} = 1$  is  
 (a)  $144\pi$  (b)  $12\pi$  (c)  $9\pi$  (d)  $16\pi$
5. Write the order and degree of the differential equation  $4\left(\frac{d^2y}{dx^2}\right)^2 = \left(\frac{dy}{dx}\right)^{7/2}$  is  
 (a) 2, 1 (b) 2, 2 (c) 2, 4 (d) 2, 5
6. Write the solution of the differential equation  $(3x^2 + 1)\frac{dx}{dy} = x^3 + x$  is  
 (a)  $\log(3x^2 + 1) = y + c$  (b)  $\log|x^3 + x| = y + c$  (c)  $\frac{x^3}{4} + x^2 = c$  (d) none
7. The degree of the differential  $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ , is  
 (a) 3 (b) 2 (c) 1 (d) not defined.
8. The number of arbitrary constants in the particular solution of a differential equation of third is  
 (a) 3 (b) 2 (c) 1 (d) 0
9. The integrating factor of the differential equation  $x\frac{dy}{dx} - y = 2x$ , is  
 (a)  $e^{-x}$  (b)  $e^{-y}$  (c)  $1/x$  (d)  $x$
10. The vector  $\cos\alpha\cos\beta\vec{i} + \cos\alpha\sin\beta\vec{j} + \sin\alpha\vec{k}$  is a  
 (a) null vector (b) unit vector (c) constant vector (d) none
11. Find the values of  $x$  and  $y$  so that the vectors  $2\vec{i} + 3\vec{j}$  and  $x\vec{i} - y\vec{j}$  are equal.
12. ABCD is a parallelogram with AC and BD as diagonals. Then find  $\vec{AC} - \vec{BD}$ .
13. Find the projection of vector  $\vec{i} + \vec{j} + \vec{k}$  on vector  $\vec{j}$ .
14. If  $\vec{a}$  and  $\vec{b}$  represents the diagonals of a rhombus, then find  $\vec{a} \cdot \vec{b}$ .
15. If a line makes an angle  $\frac{\pi}{3}, \frac{\pi}{4}$  with x-axis and y-axis respectively, then find the angle made by the line with z-axis.
16. The angle between the planes  $2x - y + z = 6$  and  $x + y + 2z = 3$  is \_\_\_\_\_
17. The distance between the planes  $2x + 2y - z + 2 = 0$  and  $4x + 4y - 2z + 5 = 0$  is \_\_\_\_\_

18. The equation of the plane which cuts equal intercepts of unit length on the co-ordinates axes is \_\_\_\_\_
19. Find the intercepts of the plane  $4x+2y-3z+24=0$ .
20. Write the unit normal to the plane  $x-3y+5z = 7$ .

### SECTION-B

**Answer the following:**

21. Find the integral  $\int (2x - \cos 4x) dx$ . **(OR)**

Integrate  $\int_{-\pi/2}^{\pi/2} \sin^3 x dx$ .

22. Verify that the function  $y = e^{-3x}$  is a solution of the differential

equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ . **(OR)**

Find the integrating factor of the differential equation  $x \frac{dy}{dx} + 2y = x^2$ .

23. Find the direction cosines of the vector joining the points A(1, 2, -3) and B(-1,-2,1), directed from A to B.
24. If  $\vec{a} = 5\vec{i} - \vec{j} - 3\vec{k}$  and  $\vec{b} = \vec{i} + 3\vec{j} - 5\vec{k}$ , then show that the vectors  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  are perpendicular.
25. Find the angle between the vectors  $\vec{i} - 2\vec{j} + 3\vec{k}$  and  $3\vec{i} - 2\vec{j} + \vec{k}$  (use dot product)
26. The Cartesian equation of a line is  $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$ . Find the vector equation of a line.

## SECTION-C

**Answer the following:**

27. Integrate :  $\int \sqrt{\sin 2x} \cos 2x \, dx$ .
28. Find  $\int \frac{3x-2}{(x+1)^2(x+3)} dx$  (OR)  $\int e^x \sin x \, dx$ .
29. Find the area of the region bounded by the ellipse  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ .
30. Show that the differential equation  $(x-y)\frac{dy}{dx} = x+2y$  is homogeneous and solve it.
31. Find the shortest distance between the lines  $\vec{r} = \vec{i} + \vec{j} + \lambda(2\vec{i} - \vec{j} + \vec{k})$  and  $\vec{r} = 2\vec{i} + \vec{j} - \vec{k} + \mu(3\vec{i} - 5\vec{j} + 2\vec{k})$ .
32. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane  $2x - 3y + 4z - 6 = 0$ . (OR)  
Find the equations of the planes that passes through three points  $(1, 1, -1)$ ,  $(6, 4, -5)$ ,  $(-4, -2, 3)$ .

## SECTION-D

**Answer the following:**

33. Find the area lying above x-axis and included between the circle  $x^2 + y^2 = 8x$  and inside of the parabola  $y^2 = 4x$ . (OR)  
Find the area of the region  $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$
34. Find the area bounded by the curves  $y = x^3$  and  $y = x$ .
35. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs.1000 double itself? (OR)  
Solve:  $\frac{dx}{dy} + y \cot x = 2x + x^2 \cot x$  ( $x \neq 0$ ) given that  $y = 0$  and  $x = \frac{\pi}{2}$ .
36. Find the equation of the plane which is perpendicular to the plane  $5x + 3y + 6z + 8 = 0$  and which contains the line of intersection of the planes  $x + 2y + 3z - 4 = 0$  and  $2x + y - z = -5$ .